Suppose p = 300,000 psi as determined from Equation (44) for $\alpha_r = 0.5$ and $\sigma_1 = 300,000$ psi. Then from Figure 48, K must be 9.0 for $k_1 = 1.75$ and N = 5 if $\sigma = 210,000$. Thus, the multiring cylinder must be quite large in size to support maximum repeated pressures.

The interferences Δ_n and residual pressures q_n have yet to be determined for the multiring container. Since the liner and the outer rings are assumed to be made from two different materials, thermal expansions must be included in the interference calculations. It is assumed that no thermal gradients exist; all components reach the same temperatures uniformly. Therefore, the interference required between the liner and the second cylinder is expressed as

$$\frac{\Delta_1}{r_1} = -\frac{u_1(r_1)}{r_1} + \frac{u_2(r_1)}{r_1} - \alpha_1 \Delta T + \alpha_2 \Delta T \qquad (49)$$

where

Δ_1 = manufactured interference

$u_1(r\ )$ = radial deformation of liner at r_1 due to residual pressure q_1 at r_1

$u_2(r_1)$ = radial deformation of Cylinder 2 at r_1 due to residual pressures q_1 at r_1 and q_2 at r_2

α = coefficient of thermal expansion at temperature

ΔT = temperature change from room temperature.

The interferences Δ_n required between the outer cylinders is again given by Equation (33) for $n \geqq 2$. The residual pressures q_n needed in calculating the Δ_n are found from Equation (32) for p_n given by Equations (46) and (38). In the calculation of the u_n from Equation (14a), the values of the moduli of elasticity, E_n at temperature should be used.

The container designed for use at temperature will have residual pressures q_n^* at room temperature different from the q_n necessary at temperature. The q_n^* are found as follows: the u_n^* are first expressed in terms of q_n^* from Equation (14a) using the values of E_n at room temperature, the Δ_n are expressed in terms of the u_n^* from Equations (49) and (33) for $\Delta T = 0$. This procedure gives the following system of equations in the q_n^*:

$$A_{11} q_1^* + A_{12} q_2^* = E_2 \frac{\Delta_1}{r_1}$$

$$A_{nn-1} q_{n-1}^* + A_{nn} q_n^* + A_{nn+1} q_{n+1}^* = E_n \frac{\Delta_n}{r_n}, \quad n = 2, 3, \ldots, N-1 \qquad (50a, b, \ldots)$$

where

$$A_{11} = \frac{k_2^2 + 1}{k_2^2 - 1} + \nu + \frac{E_2}{E_1} \left(\frac{k_1^2 + 1}{k_1^2 - 1} - \nu \right), \quad A_{nn-1} = \frac{-2}{k_n^2 - 1}, \quad A_{12} = \frac{-2k_2^2}{k_2^2 - 1},$$

$$A_{nn} = \frac{k_n^2 + 1}{k_n^2 - 1} + \frac{k_{n+1}^2 + 1}{k_{n+1}^2 - 1} = 2\frac{k_n^2 + 1}{k_n^2 - 1} \qquad A_{nn+1} = \frac{-2k_{n+1}^2}{k_{n+1}^2 - 1} = -2\frac{k_n^2}{k_n^2 - 1}$$

and where Δ_1 and the Δ_n, $n \geqq 2$ have been previously calculated for $\Delta T \neq 0$. There are N-1 linear equations (50a,b, ...) in N-1 unknowns q_n, $n = 1, 2, \ldots, N-1$ ($Q_N = 0$). These are easily solved by matrix solution on the computer.

Having calculated the residual pressures q_n^* at room temperature the residual stresses can be calculated from Equations (13a-c). These residual stresses can then be checked in order to ensure that they are within tolerated bounds. Examples of such calculations are described later when specific designs are considered. Next, the ring-segment container is considered.

Ring-Segment Container

A ring-segment container has been shown in Figure 39b. For this design, the equilibrium requirement, Equation (21), relates p_1 and p_2. Under shrink-fit it is assumed that the segments just barely contact each other, i.e., the segments carry no hoop stress. (If the segments were in strong contact with each other, they would act like a complete ring, i.e., they would carry compressive hoop stress, and the distinction between a ring-segment container and a multiring container would be lost.) Thus, the same equilibrium requirement applies to the residual pressures q_1 and q_2. This requirement is

$$p = p_1/k_2, \qquad q = q_1/k_2 \tag{51a, b}$$

In order to determine the pressures p_1 and q_1 the following radial deformation equation is formulated:

$$u_2(r_2) - u_2(r_1) + \Delta_{12} + \alpha_2 \Delta T\,(r_2 - r_1) = u_3(r_2) - u_1(r_1) + \alpha_3 \Delta T r_2 - \alpha_1 \Delta T r_1 \tag{52}$$

where

Δ_{12} = the manufactured interference defined as the amount $(r_2 - r_1)$ of the segments exceeds $(r_2 - r_1)$ of the cylinders

$u_n(r_m)$ = the radial deformation of component n at r_m due to pressure p_n or q_n at r_n and p_{n-1} or q_{n-1} at r_{n-1}

α_n = thermal coefficient of expansion of component n

ΔT = temperature change from room temperature.